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Cosmic ray showers produced by muon bremsstrahlung

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Abstract. A number of groups have observed air showers in the atmosphere at large zenith angles, where the frequency of conventional air showers induced by primary nucleons is expected to be very small. In the present work the possibility of explaining the showers in terms of muon bremsstrahlung is examined; the expected shower size spectrum for this process is calculated and the method enables the sensitivity of shower frequency to high-energy muon intensity to be examined rather simply.

Some problems associated with comparison of prediction with the experimental results are indicated. A tentative comparison indicates that when allowance is made for the effect of uncertainties in recorded shower size it is likely that the majority of the showers can be explained by muon bremsstrahlung. The possible exception is some of the events recorded by the Tokyo group in 1970 (Nagano *et al.*) which appear to have a higher penetrating particle content than would be expected on the bremsstrahlung hypothesis.

1. Introduction

Over the last few years experimental groups in Tokyo, Kiel and Durham have observed air showers incident at large angles to the vertical (above 60° or so). At these angles the flux of conventional showers due to primary protons is very small and some other mechanism is necessary for their production.

The most likely explanation would appear to be that they are due to bremsstrahlung produced in the atmosphere by energetic muons, in which case, assuming muons to be derived in the main from pions, the frequency of the biggest showers (due to muons of $E_\mu \gg 1000$ GeV) should increase as sec θ^*, θ^* being the zenith angle in the region of the pion-producing layers (see for example Wolfendale 1969). Furthermore, the size spectrum should follow an easily calculable line. However, calculations by the Tokyo group (Matano *et al.* 1968, Hara *et al.* 1969, Nagano *et al.* 1970) give a frequency that is considerably smaller than is observed and it has been proposed that the nuclear interaction cross section of the muon is much higher than given by conventional theory. Alternatively, Gawin *et al.* (1970) have proposed that the X process of Bergeson *et al.* (1968) is responsible for the large-angle showers (as well as muon-poor showers and the anomalous distribution of single energetic muons observed by the Utah group). The X process is also of relevance to the angular distribution of the showers even if they arise from muon bremsstrahlung, because if the X process is accepted then the showers should be nearly isotropic and not follow the sec θ^* 'law' expected on the conventional theory.

An excess over conventional expectation has also been observed by the Durham group (Alexander *et al.* 1969) but these workers have pointed to problems associated with an explanation in terms of inelastic muon interactions and have suggested that their conversion from measured electron densities to shower size may not be accurate.

The Kiel group (Bohm *et al.* 1969, 1970) have given an approximate frequency

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of detected showers above a particular size, 10^4, and have examined the angular distribution of the events. Their data so far, however, are not sufficient to distinguish between the isotropic and 'sec θ' angular distributions.

The three experiments referred to give somewhat different results on the angular distribution of the showers (although the differences may not in fact be significant) but they agree in finding an excess over that predicted by a straightforward calculation of the expected muon bremsstrahlung size spectrum. In what follows, the problem of calculating this expected spectrum is considered in some detail, bearing in mind a number of uncertainties which arise when conversion is made to the form in which comparison with experimental data is possible.

2. Shower size spectrum for conventional muon interactions

2.1. *Single muon spectrum*

There is the usual problem that the single muon spectrum has only been measured directly to a little below 1000 GeV and at higher energies recourse must be made to the depth–intensity relation and an assumed range–energy relation for muons. Many

Table 1. Adopted differential muon intensities in the vertical direction at sea level (see § 2.1. for description)

E_μ(GeV)	10^3	3×10^3	10^4	3×10^4	10^5
Spectrum D-70	$1{\cdot}4\times10^{-10}$	$2{\cdot}7\times10^{-12}$	$3{\cdot}6\times10^{-14}$	$6{\cdot}7\times10^{-16}$	$9{\cdot}0\times10^{-18}$
Spectrum E	$1{\cdot}1\times10^{-10}$	$2{\cdot}0\times10^{-12}$	$2{\cdot}3\times10^{-14}$	$3{\cdot}5\times10^{-16}$	$3{\cdot}6\times10^{-18}$

workers have made estimates in this way, but for the degree of accuracy needed in the present work the differences are not serious. Calculations have been made for two muon spectra, denoted by 'Spectrum D-70' and 'Spectrum E'. The first is that given by Kiraly and Wolfendale (1970), which referred to energies below 10^4 GeV, extrapolated to higher energies with the same slope ($-2{\cdot}6$, integral). This spectrum probably represents an upper limit to the allowable muon spectrum, certainly above 10^4 GeV where the assumption of no change in slope means that there must be a change in the characteristics of the interactions in which the parents of the muons are produced—a fact that follows from the well known steepening of the primary nucleon spectrum at energies above 3×10^6 GeV. 'Spectrum E' refers to the muon spectrum that would be expected if the multiplicity of muon parents (pions) varied as $E_p{}^{1/4}$ throughout, and thus the steepening of the primary spectrum is reflected in the muon spectrum.

2.2. *Calculation of expected shower size spectrum*

It is useful to present the calculation in a form in which the relevance of the various factors can be clearly seen, as follows.

Let the integral muon spectrum in the vertical direction be given by the relation $I_\mu(>E, 0) = CE^{-\gamma}$ where γ is constant (or a slowly varying function of E). As a first step we calculate $I_{h\nu}(>E, 0)$, the spectrum of vertical bremsstrahlung photons produced in 1 g cm^{-2} of air. In the second step $I(>n, 0)$ the vertical bremsstrahlung-initiated shower size spectrum is determined, and finally in the third step an expression is found for $I(>n, >\theta)$—the integral size spectrum above a given zenith angle.

The vertical bremsstrahlung photon spectrum can be expressed as

$$I_{h\nu}(>E, 0) = b_{\mathrm{rad}} K_\gamma I_\mu(>E, 0)$$

where b_{rad} is the bremsstrahlung term in the energy loss formula of muons ($b_{\mathrm{rad}} \simeq 1\cdot5 \times 10^{-6}\ \mathrm{g^{-1}\ cm^2}$ in air) and K_γ is the factor describing the reduction of the photon production spectrum as compared with a totally inelastic production of single photons by the muons with the same value of b_{rad}. As a good approximation it can be assumed that the differential cross section of bremsstrahlung production depends only on the fractional energy V and is proportional to $V^{-1}(V^2 - \frac{4}{3}V + \frac{4}{3})$; in that case

$$K_\gamma = \int_0^1 V^{\gamma-1}(V^2 - \tfrac{4}{3}V + \tfrac{4}{3})\,\mathrm{d}V.$$

K_γ is a slowly varying function of γ (e.g. for $\gamma = 2$, $K_\gamma = 0\cdot47$, and for $\gamma = 3$, $K_\gamma = 0\cdot31$) and is insensitive to small changes in the differential cross section.

The size spectrum of vertical sea level showers initiated by the above photon spectrum can be expressed in terms of $E_n(x)$, the initial photon energy necessary to give rise to a shower of n electrons at a distance x (as fluctuations in shower development are neglected at this stage, $E_n(x)$ is a well defined function):

$$I(>n, 0) = \int_0^\infty I_{h\nu}(>E_n(x), 0)\,\mathrm{d}x = I_{h\nu}(>n\ \mathrm{GeV}, 0)\int_0^\infty [n/E_n(x)\ \mathrm{GeV}]^\gamma\,\mathrm{d}x$$

and by introducing the notation

$$\int_0^\infty [n/E_n(x)\ \mathrm{GeV}]^\gamma\,\mathrm{d}x = L_\gamma(n)$$

we obtain

$$I(>n, 0) = L_\gamma(n)\,I_{h\nu}(>n\ \mathrm{GeV}, 0).$$

$L_\gamma(n)$ is a slowly varying function of γ and can be interpreted as the effective thickness of air for producing showers with one particle per GeV at the observation level. It has been derived from the one-dimensional shower calculations of Snyder (1949) by numerical integration.

In inclined directions the enhancement of the differential muon spectrum can be expressed by a factor $F_\mu(E, \theta)$, where θ is the zenith angle (as has already been mentioned, $F_\mu(E, \theta) \simeq \sec\theta^*$ for $E \gg 1000$ GeV). Since, above a few hundred GeV, $F_\mu(E, \theta)$ is a slowly changing function of E while the energy spectrum of muons changes much faster, the enhancement factor of the integral spectrum is $F_\mu(>E, \theta) \simeq F_\mu(E, \theta)$. Furthermore, as air showers of size n are preferentially produced by muons with $E \simeq n$ GeV, the enhancement for air showers is $F(>n, \theta) \simeq F_\mu(>n\ \mathrm{GeV}, \theta)$.

By introducing $F(>n, >\theta)$, that is, the average enhancement factor above a certain zenith angle, the integral size spectrum for showers with zenith angles above θ can be expressed in the following way:

$$I(>n, >\theta) = r_\gamma(>n, >\theta) I_\mu(>n\ \mathrm{GeV}, 0)$$

where

$$r_\gamma(>n, >\theta) = b_{\mathrm{rad}} K_\gamma L_\gamma(n) F(>n, >\theta).$$

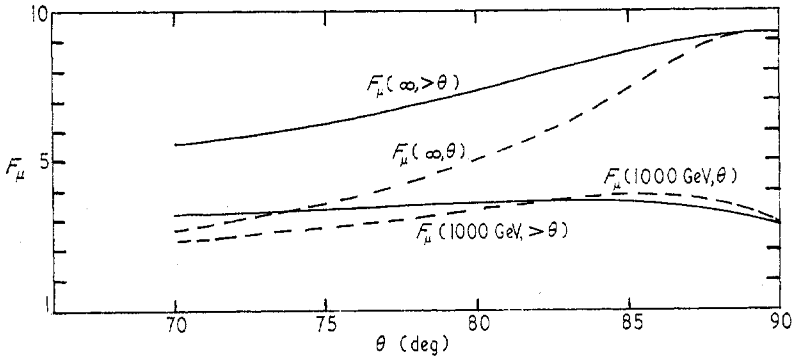

Figure 1. Muon enhancement factors for sea level zenith angle θ and energy E_μ: $F(\theta, E_\mu)$; and averages above θ: $F(>\theta, E_\mu)$. The results are given for $E_\mu = 1000$ GeV and $E_\mu \to \infty$.

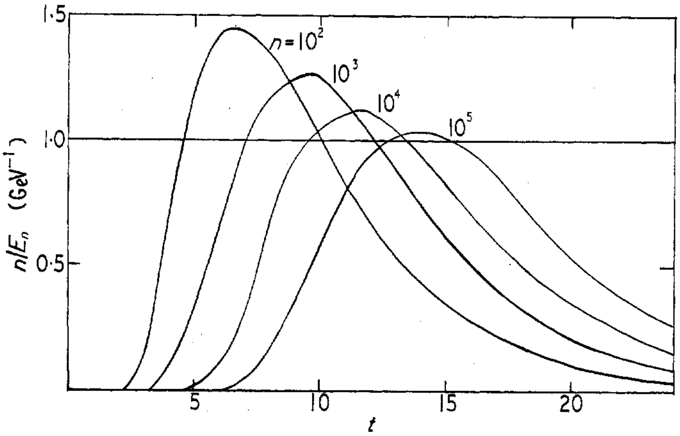

Figure 2. The adopted shower development curves (after Greisen 1956) plotted as n/E_n (GeV^{-1}) against depth. E_n is the energy of the initiating photon in GeV; t is in radiation lengths. x (g cm^{-2}) $= 37{\cdot}7t$.

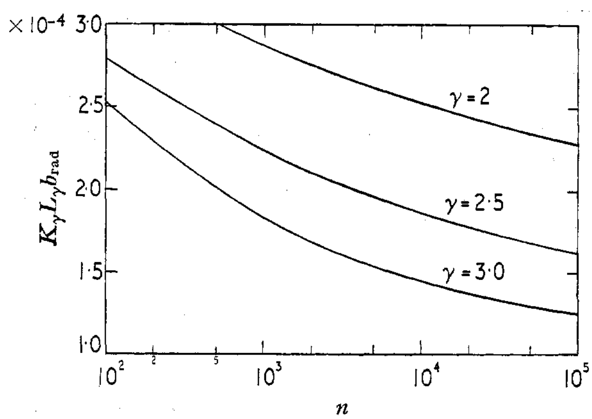

Figure 3. The factor $r_\gamma(>n, 0)$ relating the calculated integral shower size spectrum to the integral muon intensity above n GeV in the vertical direction.

The angular factor is given in figure 1 and $n/E_n(x)$ is shown in figure 2. Figure 3 gives the ratio $r_\gamma(>n, 0)$, that is, the factor by which the integral vertical muon intensities above n GeV must be multiplied in order to find the expected shower size spectrum in the vertical direction. For large-angle-size spectra the results are to be multiplied by the enhancement factors from figure 1.

In the above treatment the following two simplifying assumptions have been used:

(i) there are no fluctuations in the shower development

(ii) the longitudinal development of showers is that given by the one-dimensional theory, that is, the age of the shower is constant at a given level.

In our opinion the errors caused by these approximations are not very important in view of the large experimental uncertainties; their magnitudes are not more than a few tens of per cent and they have opposite directions.

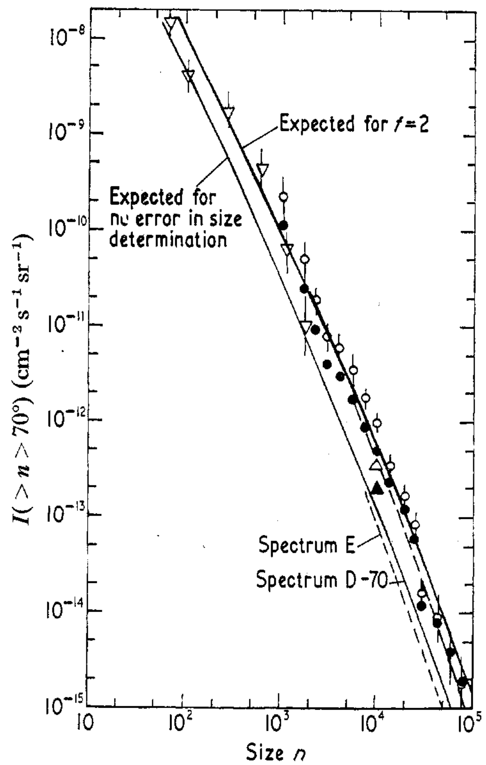

Figure 4. The predicted integral shower size spectra for $\theta > 70°$ for the two muon spectra. Also shown are the expected spectra where sizes are determined with an accuracy of a factor 2 (standard deviation). The significance of the experimental points is discussed in the text. ▽ Durham scaled to $s \simeq 1$; ○ Tokyo scaled to $s \simeq 1$; ● Tokyo raw data; △ Kiel.

The integral shower size spectra assuming $b_{\text{rad}} = 1{\cdot}5 \times 10^{-6}$ g^{-1} cm^2 (a figure which comes from the calculations of Erlykin 1965) are shown in figure 4 for the two spectra and $\theta > 70°$. At small sizes a contribution from knock-on showers has been included; this amounts to 30% at $n = 100$.

2.3. *Effect of uncertainties in shower size*

A variety of factors conspire to produce errors in the sizes allocated to individual showers in actual experiments and it is necessary to examine their effect. Often the uncertainties are symmetrical on a logarithmic scale and, if a Gaussian distribution in the logarithmic measured size about the true logarithmic value of standard deviation d ($d = \ln f$, f being the 'uncertainty factor') is assumed, then it is easy to show (e.g. Murzin and Sarycheva 1968) that the measured intensities are increased by the factor $F_{d,\gamma} = \exp(\gamma^2 d^2/2)$ (for γ varying slowly with size). Values of $F_{d,\gamma}$ are given in table 2 and the result of applying the factor to the case of $\theta > 70°$ is shown in figure 4 for what is probably a common situation in practice: $f \doteq 2$.

Table 2. Factors $F_{d,\gamma}$ by which burst intensity is increased when allowance is made for size uncertainties of magnitude f (see § 2.3)

γ \ f	1·3	1·5	2·0	3·0
2·50	1·24	1·67	4·47	43·5
2·65	1·27	1·78	5·40	69·5
2·80	1·31	1·90	6·50	113
3·0	1·36	2·10	8·60	230

3. Experimental measurements of the shower size spectrum at large zenith angles

3.1. *Comparison of the experimental arrays*

The Tokyo array has an area of several thousand square metres and is mainly in the horizontal plane, whereas the detectors at Kiel (105 m^2) and Durham (34 m^2) are arranged vertically. In the Kiel and Tokyo arrays, particle densities are recorded in the various detectors for each event and the shower axis is determined in the standard manner; the result is that the acceptance area increases with shower size, the number of axes falling outside the array area increasing with size, and the measurements consequently extend to comparatively large sizes: $n \sim 10^4$ to 10^5. In the Durham experiment, however, an attempt was made to select only those showers whose axes intersected a fixed area within the detection array; in consequence the showers collected were restricted to small sizes, $n < 2 \times 10^3$.

3.2. *General discussion of size measurements*

In the calculations reported in § 2 no allowance was made for the effect of any experimental shower selection bias. The calculations were essentially one-dimensional in character and the effect of the rapidly falling muon energy spectrum was such that the bulk of the showers contributing to the intensity for a particular size threshold were close to their maximum development, that is, the shower size parameter $s \simeq 1$. In practice, however, with the small arrays used, the density near the axis is greater. For example, in the Tokyo experiment estimates of s have been made for the detected showers and, although the authors point out that individual values are imprecise, the mean for all the showers is probably accurate; for $n > 10^4$ particles they find $\bar{s} \simeq 0{\cdot}75$.

In point of fact, the significance of 's' needs to be examined in view of the fact that at any particular level in the shower the effective age parameter varies with radial distance r (Nishimura and Kamata 1950). For example, with $n = 10^4$ and in the region of the shower maximum (11 radiation lengths), s is successively 1·3, 1·08 and 0·95 at $r = 1$, 10 and 100 m. Thus, in so far as the measurements of density in the three experiments refer in the main to radial distances somewhat below 10 m, the measured value of s will be a little larger than the mean value for the whole shower. In what follows, the variation of s with r will not be considered further, and by 's' will be meant the average value over the region of r contributing experimentally. However, in later, more precise work—both experimental and theoretical—the r dependence of s will need to be taken into account in a more systematic fashion.

When comparing the observed size spectrum with that predicted, two main effects need to be considered:

(i) the uncertainty in the individual size determination, that is, the extent to which the calculated size differs from the true size at a particular level of detection. This uncertainty comes from errors in core location, fluctuations in density in the detectors, effects of photons in the showers, etc. (to be referred to as 'recording fluctuations'). The magnitude of the increase in observed intensity over expectation arising from such uncertainties was considered in § 2.3.

(ii) the relationship between the detected size n_D and the size at maximum development n_m to which the predictions refer. In making a conversion from n_D to n_m (i.e. from $s = \bar{s}$ (detected) to $s \simeq 1$) two factors must be considered: firstly the straightforward effect of differences in age, and secondly the difference in the magnitude of the statistical fluctuations in particle number at the two values of s (these fluctuations, which come from fluctuations in longitudinal development, will be referred to as 'fundamental fluctuations').

3.3. *The measurements at Durham* (*Alexander et al.* 1969)

The experimental data will be considered in turn. The data from the Durham experimenta have been mentioned briefly already in § 3.1. They cover a region of size where, if bremsstrahlung is the production mechanism, the relevant muon energies are mainly below a few TeV and the energy spectrum is quite well known. Thus, since it is unlikely that any new process is operative in this energy region, one would expect agreement with theoretical prediction. In the preliminary report of the experiment, given by Alexander *et al.* (1969), agreement was not in fact observed at all sizes, the observed intensities being somewhat higher than those expected, for $2 \times 10^2 < n < 2 \times 10^3$. The results of a more recent analysis will now be given.

The lateral distributions of detected electrons were, as expected, somewhat steeper than those for a mean value of $s = 1$ and, in fact, corresponded to an average value of s of approximately 0·6. Thus, the observed sizes were scaled up by the appropriate factor ($\simeq 2$) to correspond to $s \simeq 1$ and hence be comparable with prediction. This procedure still stands but a recalculation of the acceptance area times solid angle has been made and the angular limit for acceptance of the events has been raised to correspond to those of the other experiments: $\theta > 70°$. The modified data are given in figure 4, events from the North and South being grouped together to improve the statistical precision. It will be noted that the new points are a little lower.

Consideration must now be given to the uncertainty in measured shower size. It is considered that this is about a factor of 2 ($f = 2$, see § 2.3) and the effect of the enhancement for this uncertainty is also shown in figure 4. It is seen that there is now no clear evidence for a significant deviation of experiment and theory.

3.4. *The results of the Tokyo group* (*Matano et al.* 1968, *Hara et al.* 1969, *Nagano et al.* 1970)

These measurements are of more interest because of their greater statistical precision and the fact that they extend to considerably larger shower sizes.

The experimental points given in the latest work (Nagano *et al.* 1970) are given in figure 4 (shown as full circles) where the predictions of the present calculations are also shown. Unlike in the work of Alexander *et al.*, no correction to $s = 1$ was made and the points therefore represent the shower sizes as detected. Basic data have been given which refer to the detected showers having $n > 10^4$ and from these the mean value of s can be determined: as mentioned already, it is 0·8 for $n > 3 \times 10^4$ and 0·75 for $n > 10^4$. An estimate of the necessary conversion to the data points to allow them to be compared with prediction comes from data given by Nagano *et al.* on the acceptance (area times solid angle above 70°) against showers size for $s = 0{\cdot}6$ and $s = 1{\cdot}0$. Allowing for the differing triggering requirements for successive size ranges we estimate that the upward correction to the points are of magnitude 2·0, 2·0, 1·8, 1·3 and 0·8 for threshold sizes 10^3, 3×10^3, 10^4, 3×10^4 and 10^5 respectively (in fact these corrections are somewhat smaller than we should have anticipated from the change in mean s).

Corrected values of the intensity are given in figure 4 as open circles and the quoted errors have been transferred to these points.

Before comparison with prediction is made some comments about the magnitude of the fluctuations in detected particle numbers are necessary. As is well known, when $s \simeq 1$ the fluctuations are near Poissonian and for the value of n encountered here their effect is small. For $s < 1$, however, the fundamental fluctuations increase and Nagano *et al.* (following Nagano 1970) adopt the Furry distribution. Their reason is not only that the detected showers are in their early stages but that fluctuations in detected numbers of electrons in individual detectors are large (i.e. some contribution from recording fluctuation effects is included). Nagano *et al.* conclude that the effect of these fluctuations is to increase the detected shower sizes by a factor of 2. So far as the fundamental fluctuations are concerned it seems to the present authors that they are significantly less than that given by the Furry distribution, but the fact remains that the net effect of fluctuations of both types is to cause a displacement of the predicted curve to larger sizes by at least a factor of two. This follows from the fact that the quoted uncertainty in shower size (arising from recording fluctuations) is a factor 2 ($f = 2$) and this corresponds to a displacement of n by about 2, as can be seen from figure 4.

The result is that, although there are still problems concerning the effect of changes of s and of fluctuations, there is no strong evidence for an excess over expectation, at least in so far as the frequencies of the showers are concerned. Where there may be an inconsistency with the bremsstrahlung explanation is in the fact that there appears to be an unusually high probability of observing a muon associated with the shower; this point is considered in more detail in § 4.

3.5. *The experiment at Kiel* (*Bohm et al.* 1969, 1970)

As with the Tokyo experiment this experiment responds to comparatively large showers, $n > 10^4$, and an intensity has been quoted for showers at angles above 70° and with $n > 10^4$. The value given by Bohm *et al.* (1969) is shown in figure 4. Presumably the age of the detected showers is similar to that in the Tokyo work so that this value should be treated in the same way. Furthermore, it is likely that the conclusion about the value of f is valid here too.

The conclusion is again that there is no evidence for a large excess although this should be qualified in view of the fact that in the later work of the Kiel group (Bohm *et al.* 1970) it is stated that the absolute value of the intensity is not certain.

4. Discussion and conclusions

From what has been said in § 2.2, the expected shower size spectrum for a particular muon spectrum can be calculated. Conversely, it is possible to work back from a measured size spectrum to the muon spectrum, assuming, that is, that the detected showers are generated by bremsstrahlung. This method may well become one of the best techniques for determining the muon energy spectrum at energies in excess of 10^4 GeV.

Such comparisons of predicted shower size with experiments as have been made suffer from uncertainties associated with lack of accurate knowledge, so far, of the distribution of true shower sizes contributing to each measured size. However, assuming that this distribution is Gaussian with a standard deviation on a logarithmic plot of a factor 2, it has been shown that near-consistency between experiment and theory results, at least for the total intensity above 70°. Thus, so far there appears to be no strong evidence against muon bremsstrahlung being responsible for at least the majority of the showers. The main experimental fact which does not fit in with the contention that *all* the showers are due to muon bremsstrahlung concerns the Tokyo results on the penetrating particle content of the showers. From the penetrating particle data Nagano *et al.* (1970) conclude that the ratio of 'nuclear cascade showers' (i.e. showers produced by the nuclear interaction of muons as distinct from bremsstrahlung) to total showers is 20–70% in the range of size $3 \times 10^3 < N < 6 \times 10^4$ and this is clearly a key observation. It is apparent that this observation must be firmly substantiated if the existence of some new nuclear interaction process is to be conclusively proved.

If muon bremsstrahlung is responsible for most of the showers, their angular distribution should fit the variation calculated using figure 1 (i.e. near sec θ^* at $E_\mu \gg 1000$ GeV). The data of Alexander *et al.* (1969) are not inconsistent with this variation but those of Matano *et al.* (1968), whilst showing an increase up to 75°, show a reduction in intensity at large angles. With the detectors used, accurate measurements at such large angles are difficult, but again if the results are substantiated then presumably another mechanism will be responsible for some of the showers. A necessary consequence would be that the muon intensity is significantly less than given by the D-70 spectrum (see § 2.1).

In conclusion, it appears that bremsstrahlung from a conventional muon spectrum together with allowance for significant uncertainties in shower size determination allows an explanation of at least most of the features of the large-angle electron showers. A clear demonstration of a new muon interaction must await substantiation of the Tokyo results on penetrating-particle content and angular distribution at the largest angles.

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